ENERGY RELATIONSHIPS AND THE COEFFICIENT OF ENERGY CONVERSION IN THE ELECTRODYNAMIC ACCELERATION OF A PLASMA, WITH CONSIDERATION OF THE PROCESSES OF MASS TRANSFER

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We examine the effect of mass-transfer produced by the physical phenomena of plasma recombination, ambipolar diffusion, and electrode erosion, as well as the effect of the resistance forces on the energy relationships and the coefficient of energy conversion in the electrodynamic acceleration of a plasma.

Complex processes of mutual transformation from one form of energy to another take place in the electrodynamic acceleration of a plasma. The ultimate goal of the electrodynamic acceleration is the conversion of the electrical energy, stored in a capacitor, into the kinetic energy of a plasma jet with specified parameters, and with the highest possible coefficient of energy conversion. The coefficient of energy conversion is understood to refer to the ratio of the kinetic energy of the plasma jet to the electrostatic energy, stored in the capacitor, and expended on the acceleration.

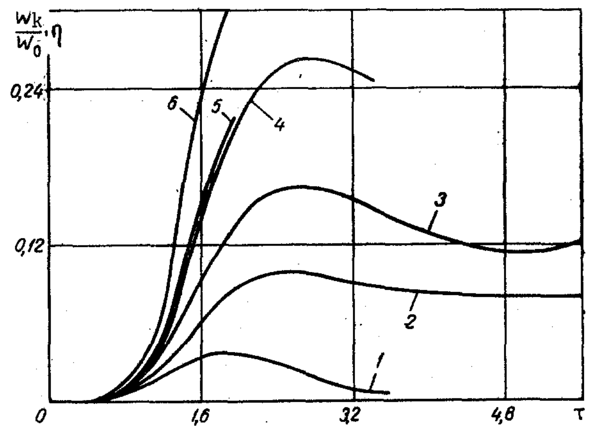

Fig. 1

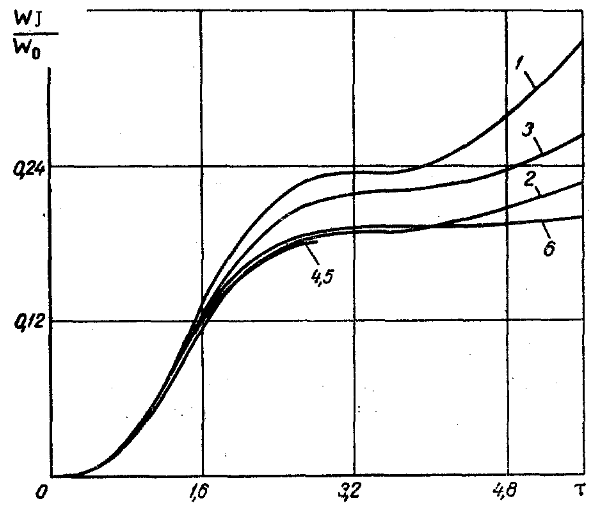

Fig. 2

Fig. 1. Change in the energy conversion factor η and in the kinetic energy W_k/W_0 during time τ for the following parameters of the acceleration systems: 1) $q = 1$, $\alpha = 0.1$, $\gamma_1 = 0$, $\gamma_2 = 0$, $\gamma_3 = 1$, $\gamma_4 = 1$, $\delta_1 = 0$, $\delta_2 = 1$, $\delta_3 = 0$, $\delta_4 = 0$; 2) $q = 1$, $\alpha = 0.1$, $\gamma_1 = 0$, $\gamma_2 = 0$, $\gamma_3 = 1$, $\gamma_4 = 1$, $\delta_1 = 0$, $\delta_2 = 0.1$, $\delta_3 = 0$, $\delta_4 = 0$; 3) $q = 1$, $\alpha = 0.1$, $\gamma_1 = 0$, $\gamma_2 = 0$, $\gamma_3 = 1$, $\gamma_4 = 0.1$, $\delta_1 = 0$, $\delta_2 = 0.1$, $\delta_3 = 0$, $\delta_4 = 0$; 4) $q = 1$, $\alpha = 0.1$, $\gamma_1 = 0$, $\gamma_2 = 0$, $\gamma_3 = 0.1$, $\gamma_4 = 0.1$, $\delta_1 = 0$, $\delta_2 = 0.1$, $\delta_3 = 0$, $\delta_4 = 0$; 5) $q = 1$, $\alpha = 0.1$, $\gamma_1 = 1$, $\gamma_2 = 0$, $\gamma_3 = 1$, $\gamma_4 = 1$, $\delta_1 = 0$, $\delta_2 = 0.1$, $\delta_3 = 0$, $\delta_4 = 0$; 6) according to [2].

Fig. 2. Change in W_J/W_0 in time τ; the parameters and the notation are the same as in Fig. 1.

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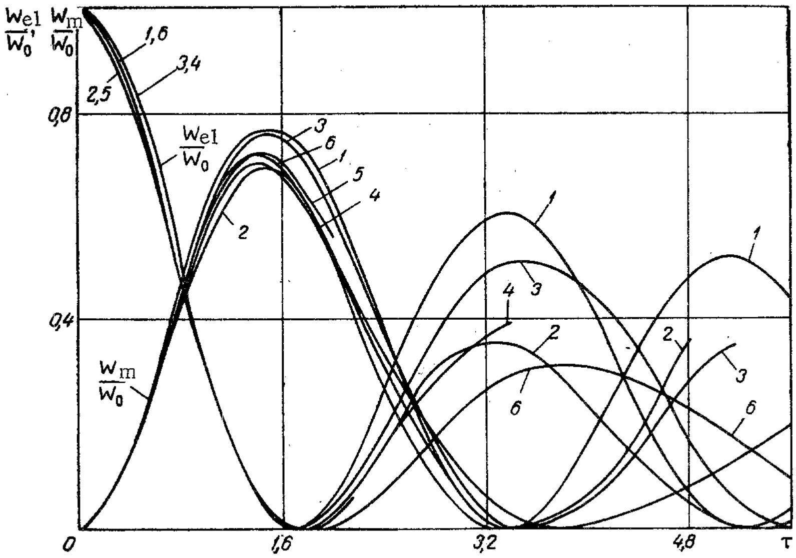

Fig. 3. Change in the electrical energy W_{el}/W_0 and in the magnetic energy W_m/W_0 during time τ; the parameters and notation are the same as in Fig. 1.

The initial energy in the electrodynamic acceleration of a plasma – with the accumulator in the form of a capacitor – is the electrostatic energy

$$W_0 = \frac{C_0 V_0^2}{2}. \tag{1}$$

If the breakdown of a gas-discharge gap obeys the law of Paschen in the slave regime a portion of the electrostatic energy (W_{ion}) is irreversibly expended on the ionization of the gas-discharge gap.

Immediately following the breakdown, as soon as current begins to flow through the circuit, the electrostatic energy is converted to magnetic energy

$$W_m = \frac{LI^2}{2}. \tag{2}$$

The currents flowing through such accelerators reach magnitudes of 100–1000 kA in impulsive discharge, as a result of which a large magnetic force begins to act on the plasma. Indeed, the magnetic force with such acceleration

$$F_m = \frac{b}{2} I^2 \tag{3}$$

and with a value of $b = 2 \cdot 10^{-7}$ H/m a current $I \simeq 300$ kA results in

$$F_m = \frac{2}{2} \cdot 10^{-7} \cdot 10^{11} = 10^4 \text{ N}.$$

The pressure applied to 1 cm^2 with such a force is

$$P = \frac{10^4}{10^{-4}} = 10^8 \frac{\text{N}}{\text{m}^2} \simeq 1.02 \cdot 10^3 \text{ atm}.$$

The great magnetic pressures, of an order of magnitude of 10^3 atm and higher, lead to intensive acceleration of the plasma to velocities of the order of 10^4–10^6 m/sec. Therefore, in electrodynamic acceleration a portion of the magnetic energy is converted to kinetic energy, i.e.,

$$W_k = \frac{mv^2}{2}. \tag{4}$$

Another portion of the magnetic energy is converted into Joule heat. Even in the case of insignificant circuit and plasma resistances of the order of 10^{-3} Ω, with the passage of high currents, a substantial portion of the energy is converted into heat:

$$W_J = \int_0^t RI^2 dt. \tag{5}$$

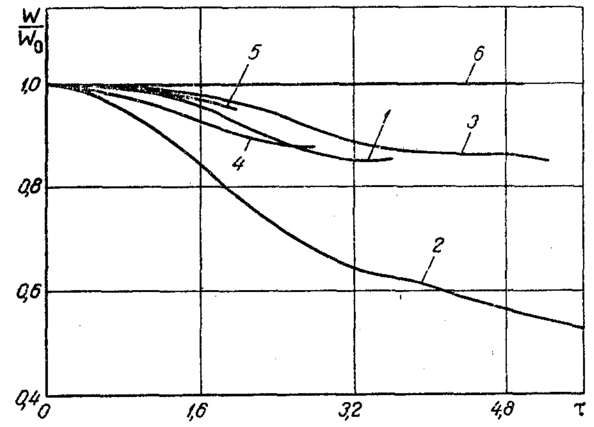

Fig. 4. Change in the total energy W/W_0 during time τ; the parameters and notation are the same as in Fig. 1.

Because of the oscillatory properties of the discharge circuit, we find reciprocal transfer of electrostatic energy into magnetic, and vice versa. A portion of the magnetic energy is spent on processes of mass and electricity transfer, on overcoming resistance, etc.

The kinetic energy of a plasma moving in the magnetic field of a flowing current, in turn, as a consequence of the Faraday law of electromagnetic induction, changes into magnetic and electric energy. Moreover, the kinetic energy is dissipated on overcoming the forces of friction and resistance, the generation of shock waves, etc.

Let us consider the energy balance in the electrodynamic acceleration of a plasma with consideration of the processes of mass transfer and the effect of the resistance forces on the basis of the data from [1], where it is demonstrated that these processes exert considerable influence on the fundamental characteristics of an electrodynamic accelerator, as described by the system of equations (6)-(10):

$$\frac{dmv}{dt} = \frac{b}{2} I^2 - F, \tag{6}$$

$$I = -C_0 \frac{dV}{dt}, \tag{7}$$

$$\frac{dLI}{dt} + RI - V = 0, \tag{8}$$

$$L = L_0 + bz, \tag{9}$$

$$\frac{dm}{dt} = -a_1 m - a_2 m^2 + a_3 |I| + a_4 I^2 \tag{10}$$

for the initial conditions

$$t = 0 \quad z = v = I = 0, \; V = V_0, \; m = m_0.$$

Here Eq. (6) describes the motion of the center of inertia for the plasma mass; Eqs. (7)-(9) describe the electromechanical processes in the electrical circuit, and Eq. (10) describes the mass balance, with consideration given to the processes under consideration.

The first term in the right-hand member of (10) describes the reduction in mass as a consequence of diffusion, while the second term describes the reduction in mass as a consequence of particle recombination; the third and fourth terms give the increase in mass which is produced by electrode erosion resulting from ion bombardment and Joule fusion of the electrodes, respectively. The proportionality factors a_1, a_2, a_3, and a_4 are determined experimentally or theoretically, on the basis of the kinetics of the elementary processes [1].

The resistance force F in (6), according to [1], can be presented in the form

$$\mathbf{F} = b_1 \mathbf{v} + b_2 \mathbf{v} m + b_3 |I| \mathbf{v} + b_4 v^2 + \ldots \tag{11}$$

and is governed by the friction of the moving plasma against the electrodes (the first term), by the processes of friction in mass transfer (the second and third terms), and by the resistance of the external medium (the fourth term). The quantities b_1, b_2, b_3, and b_4 in (11) are proportionality factors which were evaluated in [1].

The overall energy balance can be written in the form of the equation for the conservation of energy, which is the first integral of the system of equations (6)-(10):

$$\frac{C_0 V_0^2}{2} = \frac{C_0 V^2}{2} + \frac{LI^2}{2} + \frac{mv^2}{2} + R_0 \int_0^t I^2 dt + W_l. \tag{12}$$

Having divided both parts of the equation by the electrostatic stored energy W_0, we find

$$1 = \frac{V^2}{V_0^2} + \frac{LI^2}{C_0V_0^2} + \frac{2\,R_0}{C_0V_0^2}\int_0^t I^2dt + \frac{mv^2}{C_0V_0^2} + \frac{2W_l}{C_0V_0^2}\,. \tag{13}$$

It is most convenient to consider this equation in dimensionless variables, which we introduce on the basis of the formulas

$$\tau = \frac{t}{\sqrt{L_0C_0}},\quad y = \frac{b}{L_0}z,\quad y' = b\sqrt{\frac{C_0}{L_0}},$$
$$\varphi = \frac{V}{V_0},\quad \mu = \frac{m}{m_0},\quad \varphi' = \sqrt{\frac{L_0}{C_0}}\,\frac{I}{V_0}\,. \tag{14}$$

Considering (14), as well as the dimensionless quantities

$$q = \frac{b^2C_0^2V_0^2}{2m_0L_0},\quad \alpha = R\sqrt{\frac{C_0}{L_0}},$$
$$\gamma_1 = a_1\sqrt{L_0C_0},\quad \gamma_2 = a_2m_0\sqrt{L_0C_0},$$
$$\gamma_3 = \frac{a_3C_0V_0}{m_0},\quad \gamma_4 = \frac{C_0^2V_0^2}{\sqrt{L_0C_0}m_0}\,a_4, \tag{15}$$
$$\delta_1 = \frac{b_1}{m}\sqrt{L_0C_0},\quad \delta_2 = b_2\sqrt{L_0C_0},$$
$$\delta_3 = \frac{1}{2}\,\frac{m_iC_0V_0}{em_0},\quad \delta_4 = \frac{b_4L_0}{m_0b},$$

from (13) we can obtain the following equation in dimensionless form:

$$1 = \varphi^2 + (1+y)\varphi'^2 + 2\alpha\int_0^\tau \varphi'^2d\tau + \frac{\mu y'^2}{2q} + \frac{W_l}{W_0}\,. \tag{16}$$

The factor giving the conversion into kinetic energy of the electrical energy stored in the capacitor can be written in the form

$$\eta = \frac{W_k}{W_0} = \frac{\mu y'^2}{2q}\,. \tag{17}$$

Figure 1 shows the change in the energy conversion factor for the case of erosion and plasma diffusion, with consideration of the "diffusion" force of friction. Curve 1 corresponds to an intensive electrode erosion process as a consequence of the Joule fusion of the electrodes and their bombardment by an ion stream in conjunction with the strong influence of the forces of diffusion friction. The conversion factor η_1 is at its maximum in this case in the acceleration time interval $\tau = 1.6$–2.2 and does not exceed 0.03718. A tenfold reduction in the magnitude of the diffusion friction as compared with case 1 (curve 2) leads to a marked increase in the energy conversion factor ($\eta_2 = 0.1013$). The maximum for η_2 shifts toward the greater acceleration times in this case, i.e., $\tau = 2.2$–3.0. The tenfold reduction in the parameter γ_4 as compared with case 2 enables us to evaluate – at any instant of time – the effect of mass release as a consequence of the Joule fusion of the electrodes, while the reduction in the parameter γ_3 as compared with case 3 (curve 4) permits us to evaluate the effect of ion bombardment. The maximum values of η for these parameters are $\eta_3 = 0.1656$ and $\eta_4 = 0.2651$ for acceleration times of $\tau = 2.6$ and 2.8 respectively.

All other conditions being equal, intensive diffusion (curve 5) leads to a pronounced increase in the energy conversion factor.

Curve 6 has been plotted from the data in [2] for the case in which the release of mass during the plasma acceleration process is assumed constant. We see that in considering the processes of mass transfer there is a reduction in the values of η that are obtained. Thus, for $\tau = 1.6$, $\eta_1 = 0.03349$, $\eta_2 = 0.0617$, $\eta_3 = 0.09294$, $\eta_4 = 0.1377$ and $\eta_5 = 0.1408$, whereas $\eta_6 = 0.2282$.

The losses to Joule heat are defined as

$$\frac{W_J}{W_0} = 2\alpha\int_0^\tau \varphi'^2d\tau. \tag{18}$$

The integral was calculated numerically by the Simpson method, with an interval of h = 0.4. The calculation results are given in Fig. 2.

The losses to Joule heat for the cases being analyzed are approximately identical for the acceleration time interval $\tau = 0.4$ and do not exceed 0.24. With a further increase in τ these losses increase, and it is characteristic that the losses for the cited cases 1-3 are higher than for case 6.

The effect of the subject mass-transfer processes on electric (W_{el}) and magnetic (W_m) energy can be estimated from Fig. 3.

The losses governed by the mass-transfer processes are defined as

$$\frac{W_l}{W_0} = 1 - \frac{W}{W_0},$$

where

$$\frac{W}{W_0} = \frac{W_{el}}{W_0} + \frac{W_m}{W_0} + \frac{W_J}{W_0} + \frac{W_k}{W_0}.$$

They are greater for intensive Joule fusion (curve 2 in Fig. 4). However, for the remaining cases, in the acceleration time intervals of $\tau = 0$-3.0, they do not exceed 0.1460.

These calculations provide a complete picture as to the relationship between the various forms of energy and the effect of the various energy-dissipation processes and the processes of mass transfer on the transformation of one form of energy to another. Analysis shows that the enumerated processes result in a reduction, as should be the case, in the energy conversion factor. However, in practical terms, it is difficult to eliminate these processes, and from among the above-cited calculations we can seek means of reducing them, in addition to evaluating the relative effect of these processes.

NOTATION

m_i	is the ion mass of the accelerated plasma;
m	is the accelerated mass;
m_0	is the initial mass of the accelerated plasma;
z	is the coordinate of the center of inertia;
t	is the time;
I	is the current;
V	is the voltage;
V_0	is the initial voltage at the capacitor;
F	is the force of resistance to plasma motion;
C_0	is the capacitance of the capacitor bank;
R	is the total resistance of the circuit;
L	is the inductance of the power leads and of the capacitor;
b	is the distributed inductance per unit length of the coaxial line;
τ, y, y', φ, μ, φ', W_{el}/W_0, W_m/W_0, W_J/W_0, W_l/W_0, W_k/W_0	are the dimensionless quantities for time, path, velocity, voltage, mass, current, instantantaneous electrostatic energy, magnetic energy, Joule heat, energy losses on mass transfer and the force of resistance, and the kinetic energy, respectively;
q, δ_1, δ_2, δ_3, δ_4, γ_1, γ_2, γ_3, γ_4, α	are dimensionless parameters;
e	is the electron charge;
a_1, a_2, a_3, and a_4	are mass coefficients of diffusion, recombination, electrode erosion as a consequence of ion bombardment, and electrode erosion as a consequence of Joule fusion, respectively;
b_1, b_2, b_3, and b_4	are proportionality factors by means of which we take into consideration, respectively, the friction of the moving plasma against the electrodes (b_1), the friction processes in mass transfer (b_2, b_3), and the effect of the resistance of the external medium (b_4).

LITERATURE CITED

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2. P. M. Kolesnikov, Zhur. Tekh. Fiz., 34, 11, 1933 (1964).